Wheatstone Bridge and Meter Bridge

Objectives

After going through this lesson, learner will be able to :

- Interpreting Wheatstone bridge and meter bridge
- Apply Kirchhoff's rules for determining the condition for balanced Wheatstone bridge
- Use meter bridge to determine the resistance of a wire, specific resistance of the materia of a wire, verify laws of combination of resistances using a meter bridge
- Solve Problems related to meter bridge

Content Outline

- Unit Syllabus
- Module wise distribution of unit syllabus
- Words you must know
- Introduction
- Kirchhoff's rules
- Wheatstone bridge
- Simple applications of Kirchhoff's rules for condition for Balanced Wheatstone bridge
- Wheatstone bridge and meter bridge
- Application of meter bridge
- Summary

Unit Syllabus

Electric current, flow of electric charges in a metallic conductor, drift velocity and mobility, and their relation with electric current; Ohm's law, electrical resistance, V-I characteristics (linear and non-linear), electrical energy and power, electrical resistivity and conductivity.

Carbon resistors, colour code for carbon resistors; series & parallel combinations of resistors; temperature dependence of resistance.

Internal resistance of a cell, Potential difference and a emf of a cell, combination of cells in series and in parallel.

Kirchhoff's laws and simple applications; Wheatstone bridge, meter bridge.

Potentiometer- principle and its application to measure potential difference and for comparing emfs of two cells; measurement of internal resistance of a cell.

Module Wise Distribution Of Unit Syllabus

The above unit has been divided into 8 modules for better understanding.

Module 1	• Electric current, • Solids liquids and gases • Need for charge carriers speed of charge carriers in a metallic conducto • Flow of electric charges in a metallic conductor • Drift velocity, • Mobility and their relation with electric current • Ohm's law
Module 2	• Electrical resistance, • V-I characteristics (linear and non-linear), • Electrical energy and power, • Electrical resistivity and conductivity • Temperature dependence of resistance
Module 3	• Carbon resistors, • Colour code for carbon resistors; • Metallic Wire resistances • Series and parallel combinations of resistors • Grouping of resistances • Current and potential differences in series and parallel circuits
Module 4	• Internal resistance of a cell, • Potential difference and emf of a cell, • Combination of cells in series and in parallel. • Need for combination of cells
Module 5	• Kirchhoff's Rules • Simple applications. of Kirchhoff's Rules for calculating current s an voltages • Numerical
Module 6	• Wheatstone bridge • Balanced Wheatstone bridge condition derivation using Kirchhoff' Rules

	• Wheatstone bridge and Meter Bridge. • Application of meter bridge
Module 7	• Potentiometer – • Principle • Applications to • Measure potential difference • Comparing emf of two cells; • Measurement of internal resistance of a cell. • Numerical
Module 8	• Numerical • Electrical energy and power

Module 6

Words You Must Know

- **Potential drop across resistance**: It is the potential difference between ends of a resistance. In Ohm's law V = IR where, V is potential drop across resistance (for details see role of resistance in module 1)
- **Electromotive Force** (EMF): It is the force which makes change to flow in an electrical circuit. It is defined as **Work done** in moving a unit positive charge once in a closed circuit.
- **EMF of a cell (ε)**: It is the max. Potential difference between electrodes of a cell when no current is being drawn from the cell.
- **TPD of a cell (V)** - It is the max. Potential difference between electrodes of a cell when current is being drawn.
- **Internal resistance of a cell (r)**: It is the resistance offered by electrolyte to current flowing.
- **Kirchhoff's rules**: These are the tools to analyse an electrical circuit.

Two rules called Kirchhoff's rules, are very useful for analysis of electric circuits.

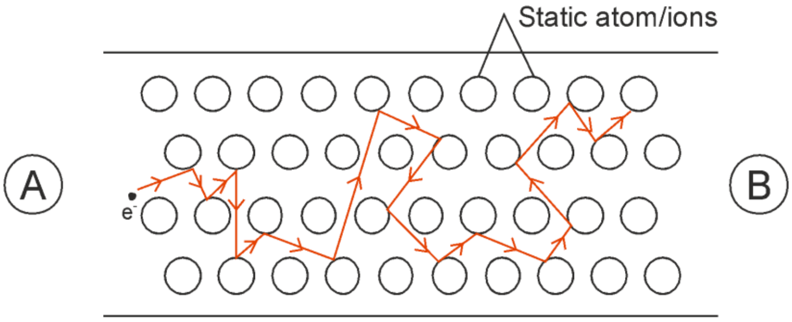
These were:

    - **Junction rule**: At any junction, the sum of the currents entering the junction is equal to the sum of currents leaving the junction
    - **Loop rule**: The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero

- **Series combination of resistances:** When the same current flows through all the devices, it is said to be a series combination.
- **Parallel combination of resistances**: When all devices have the same potential difference, it is said to be a parallel combination.
- **Electrical circuit**: It is the arrangement of electrical devices like resistance, cell, etc. to achieve a purpose /objective.
- **Bridge**: Electrical devices connected in the form of a rhombus form a bridge. There are bridges of different electrical devices Example Bridge of resistors, capacitors or diodes etc
- **Galvanometer:** It is the device which detects small currents. A Galvanometer has very high resistance.
- **Jockey**: It is a metallic rod whose one end has a knife edge which slides over the wire and the other end is connected.

Introduction

In our previous modules we have considered resistance and the cause of resistance



The Fig. shows motion of an electron within the conductor

Each metal has a large number of free electrons which are in constant motion and collide with themselves as well as static atoms or ions which fall in their way and rebound, again collide and rebound.

These collisions of electrons with static atoms or ions are the cause of resistance.

Resistance of a conductor is

i) directly proportional to length of conductor $R \propto l$

ii) inversely proportional to area of cross-section $R \propto 1/A$

Resistance $R = \rho\, l/A$

where **ρ = specific resistance/resistivity**

Factors affecting value of Resistance:-

(1) Dimensions of conductor

(2) Nature of material of conductor

Resistance of a given conductor is not fixed or constant as it depends upon dimensions of conductors.

In a circuit we may connect resistances in series and parallel with suitable source, for any desired effect.

Kirchhoff's Rules

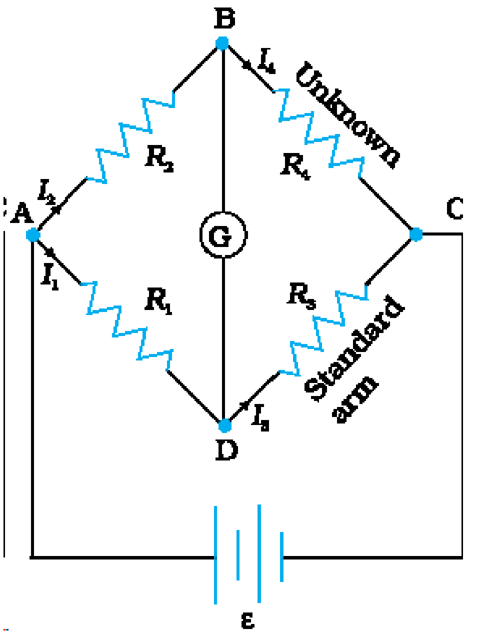
You learnt **Kirchhoff's rules** for calculating currents and voltages in a circuit. The formulae we have derived earlier for series and parallel combinations of resistors are not always sufficient to determine all the currents and potential differences in the circuit. Two rules, called Kirchhoff's rules, are very useful for analysis of electric circuits. These were

(a) **Junction rule**: At any junction, the sum of the currents entering the junction is equal to the sum of currents leaving the junction

(b) **Loop rule**: The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero

Let us now study a very special way of connecting resistances, which can be used to find values of unknown resistances

Wheatstone Bridge



The bridge has four resistors R_1, R_2, R_3 and R_4. Across one pair of diagonally opposite points (A and C in the figure) a source is connected. This (i.e., AC) is called the battery arm. Between the other two vertices, B and D, a galvanometer G (which is a device to detect currents) is connected. This line, shown as BD in the figure, is called the galvanometer arm.

For simplicity, we assume that the cell has no internal resistance. In general there will be currents flowing across all the resistors as well as a current I_g through G. Of special interest, is the case of a balanced bridge where the resistors are such that $I_g = 0$. We can easily get the balance condition, such that there is no current through G. In this case, the Kirchhoff's junction rule applied to junctions D and B

Let us see how this can be an electrical device used for finding value of unknown resistance.

Simple Applications of Kirchhoff's Rules for Condition for Balanced Wheatstone Bridge

A balanced bridge gives a relation between the four resistances R_1 , R_2 , R_3 and R_4 .

For simplicity, we assume that the cell has no internal resistance. In general there will be currents flowing across all the resistors as well as a current I_g through G.

Of special interest, is the case of a balanced bridge where the resistors are such that Ig= 0.

We can easily get the balance condition, such that there is no current through G. In this case, the Kirchhoff's junction rule applied to junctions D and B (see the figure)

Immediately gives us the relations $I_1 = I_3$ and $I_2 = I_4$.

Next, we apply Kirchhoff's loop rule to closed loops ADBA and CBDC. The first loop gives

$-I_1 R_1 + 0 + I_2 R_2 = 0 \quad (Ig = 0)$

and the second loop gives, upon using

$I_3 = I_1, I_4 = I_2$

$I_2 R_4 + 0 - I_1 R_3 = 0$

From Equations above we get: $\frac{\square_1}{\square_2} = \frac{\square_2}{\square_1}$

$$\frac{\square_1}{\square_2} = \frac{\square_4}{\square_3}$$

The derivation for the condition is based on the fact that if four resistances $\square_1$, $\square_2$, $\square_3$ □□□ $\square_4$ are connected in form of a bridge with a galvanometer (G) and a battery as shown in the figure.

If the galvanometer shows no deflection, then the bridge is said to be balanced. In balanced condition

$$\frac{\square_1}{\square_2} = \frac{\square_3}{\square_4}$$

(Product of opposite resistances is equal)

The Wheatstone bridge and its balance condition provide a practical method for determination of an unknown resistance. Let us suppose we have an unknown resistance, which we insert in the fourth arm; R_4 is thus not known. Keeping known resistances R_1 and R_2 in the first and second arm of the bridge,

We go on varying R_3 till the galvanometer shows a null deflection.

So once we have a balanced bridge

From the balance condition the value of the unknown resistance R_4 is given by:

$R_4 = R_3 \frac{\square_2}{\square_1}$

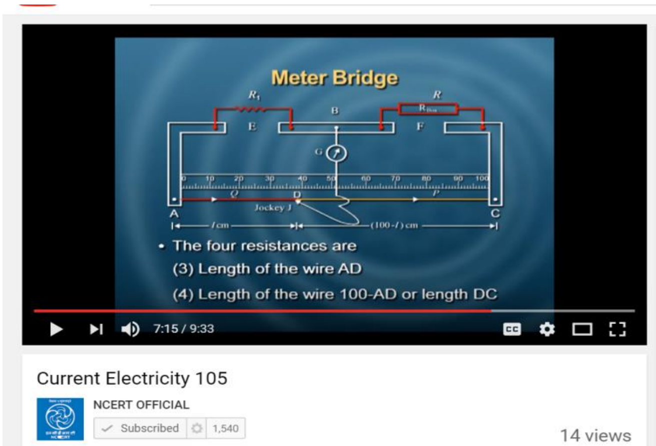
This last equation relating all the four resistors is called the balance condition for the galvanometer to give zero or null deflection.

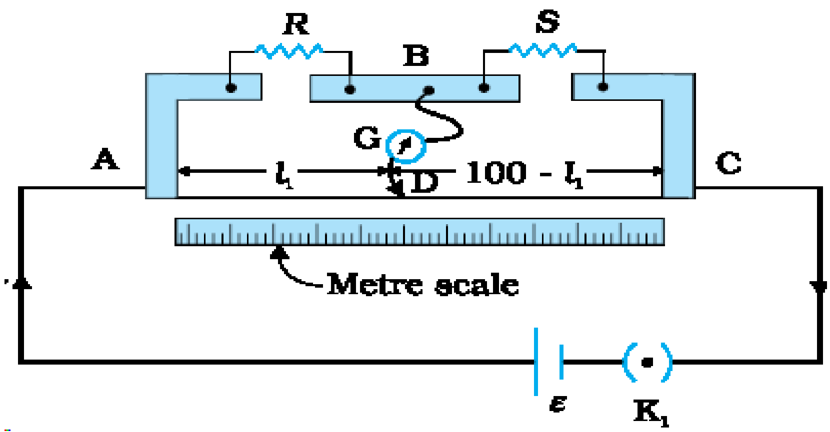
The Wheatstone bridge and its balance condition provide a practical method for determination of an unknown resistance. Let us suppose we have an unknown resistance, which we insert in the fourth arm; R_4 is thus not known. Keeping known resistances R_1 and R_2 in the first and second arm of the bridge, we go on varying R_3 till the galvanometer shows a null deflection. The bridge then is balanced, and from the balance conditions the value of the unknown resistance R_4

A practical device using this principle is called the **meter bridge.**

Wheatstone Bridge and Meter Bridge

https://www.youtube.com/watch?v=9j3hu2dO-5I&feature=youtu.be





Description of Meter Bridge

This is a simple modified design making use of a resistance wire of uniform area of cross-section and homogeneity. The meter length of the wire gives it the name meter bridge. Two resistance of the Wheatstone bridge are provided by a single wire. A jockey (movable key) can be used to find a position on the wire to give zero deflection in the galvanometer .

It consists of a wire of length 1m and of uniform cross sectional area stretched taut and clamped between two thick metallic strips bent at right angles, as shown.

The metallic strip has two gaps across which resistors can be connected. The end points where the wire is clamped are connected to a cell through a key. One end of a galvanometer is connected to the metallic strip midway between the two gaps. The other end of the galvanometer is connected to a 'jockey'. The jockey is essentially a metallic rod whose one end has a knife-edge which can slide over the wire to make electrical connection. S is an unknown resistance whose value we want to determine. It is connected across one of the gaps. Across the other gap, we connect a standard known resistance R. The jockey is connected to some point D on the wire, a distance l cm from the end A. The jockey can be moved along the wire. The portion AD of the wire has a resistance (resistance per unit centimeter) l_1, the. The portion DC of the wire similarly has a resistance (resistance per unit centimeter) x $(100-l_1)$. The four arms AB, BC, DA and CD [with resistances R, S, R_{cm} l and R_{cm} (100-l)] obviously form a Wheatstone bridge with AC as the battery arm and BD the galvanometer arm.

If the jockey is moved along the wire, then there will be one position where the galvanometer will show no current.

Let the distance of the jockey from the end A at the balance point be $l = l_1$. The four resistances of the bridge at the balance point then are R, S, $R_{cm} l_1$ and $R_{cm} (100 – l_1)$.

The balance condition,

The resistances offered by the wire sections can be given in terms of the length of the wire section , from the figure we can see if s is the unknown resistance '

$$S = R\frac{100 - l_1}{l_1}$$

Application of Meter Bridge

By choosing various values of R, we would get various values of l_1 , and calculate S each time. An error in measurement of l_1 would naturally result in an error in S. It can be shown that the percentage error in R can be minimized by adjusting the balance point near the middle of the bridge, i.e., when l_1 is close to 50 cm.

So, a practical form of Wheatstone bridge is a meter bridge

Note:

- **Thick metallic strips of copper are used to reduce resistance and to increase flow of current.**
- **S is the unknown resistance whose value determines which is inserted in the left gap.**
- **R is the standard known resistance.**

- By choosing various values of R, we would get various values of l_1, and calculate R each time. An error in measurement of l_1 would naturally result in an error in S.
- **It can be shown that the percentage error in S can be minimised by adjusting the balance point near the middle of the bridge i.e. when $\square_1$ is close to 50 cm. (This requires a suitable choice of S)**

Important points regarding meter bridge

- **Wheatstone bridge and Meter Bridge are not useful in measuring very small and very high resistances. This is because in each the balance length will be closer to one of the ends of the scale and is very difficult to measure accurately.**
- **The balance length is not affected by interchanging positions of galvanometer and battery. This is because current in the circuit remains the same so potential drop across length l_1 remains same**.
- **It is desirable to get a balance point in the middle of the wire (close to 50cm) because percentage error is reduced and meter bridge (Wheatstone bridge) is most sensitive as all the four resistances become equal.**
- **The Wheatstone bridge is said to be most sensitive when all the four resistances are equal i.e. $\square_1 = \square_2 = \square_3 = \square_4$. In this condition the Wheatstone gives most accurate value of unknown resistance**

  Let us do some examples for better understanding of the Meter Bridge and Wheatstone bridge.

Solved Examples

Make sure, you draw a rough circuit diagram each time for each problem to visualize the problem

Example

If Wheatstone bridge circuit, P=5 Ω,Q=6 Ω, =10 Ω and S=5 Ω. Find the additional resistance to be used in series of S, so that the bridge is balanced.

Solution

Let the bridge be balanced when additional resistance x be put in series of S, then

$S + X = \frac{\square}{\square}\square$

Or

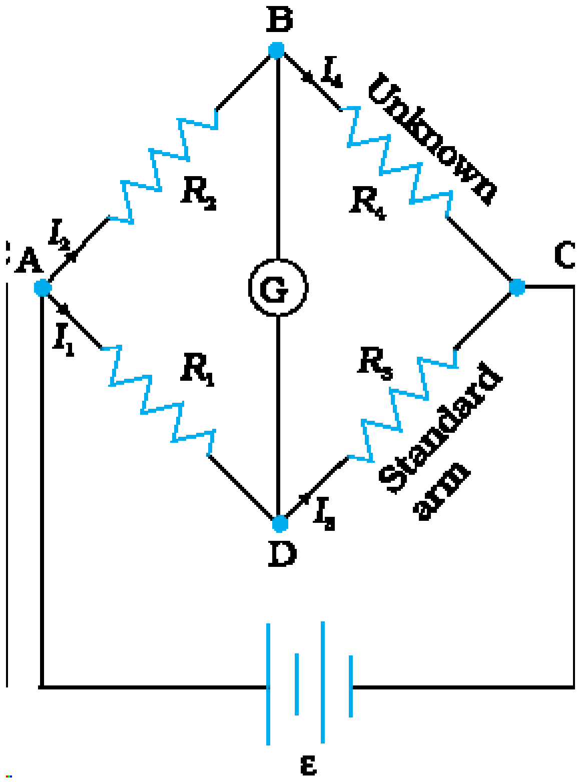
X= $\frac{□}{□}$ □ − □

= $\frac{6}{5} \times 10 - 5$

= 7 Ω

Example

The Wheatstone bridge circuit has the resistances in various arms as shown in the figure. Calculate the current through the galvanometer. (where R_1 = 100 □, R_2= 60 *Ω*, R_3= 5*Ω*, R_4=10 *Ω* and G = 15 *Ω*)



Solution

In the closed loop ABDA;

$$100□_1 + 15□_□ - 60□_2 = 0$$

Or $$20□_1 + 3□_□ - 12□_2 = 0$$

In the loop BCDB;

$$10(□_1 - □_□) - 5(□_2 + □_□) - 15□_□ = 0$$

$$10□_1 - 30□_□ - 5□_2 = 0$$

$$2□_1 - 6□_□ - □_2 = 0$$

In the closed loop ADCEA;

$$60\square_2 + 5(\square_2 + \square_\square) - 10\square_\square = 0$$

or

$$65\square_2 + 5\square_\square = 10$$

or

$$13\square_2 + \square_\square = 2$$

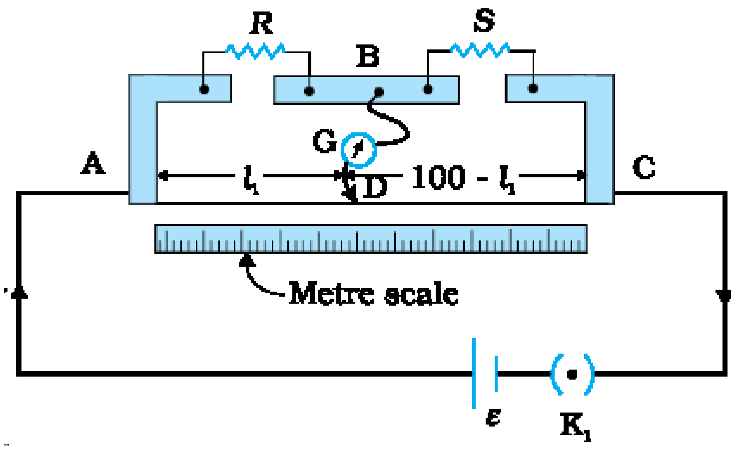
On solving (i), (ii) and (iii), we get

$$\square_\square = 4.87\ \square\square$$

Example

In a meter bridge given in figure, the null point is found at a distance of 33.7 cm from A. if now a resistance of 12 Ω is connected in parallel with S, the null point occurs at 51.9 cm. determine the values of R and S

Solution



(i)

$\frac{\square}{\square} = \frac{51.9}{48.1}$

$\square\square$

$$\frac{\square(12+\square)}{12\square} = \frac{51.9}{48.1} \text{ ... (ii)}$$

Putting the value of $\frac{\square}{\square}$ from (i) and (ii), we have

$$\frac{(12+\square)}{12} \times \frac{33.7}{66.3} = \frac{51.9}{48.1}$$

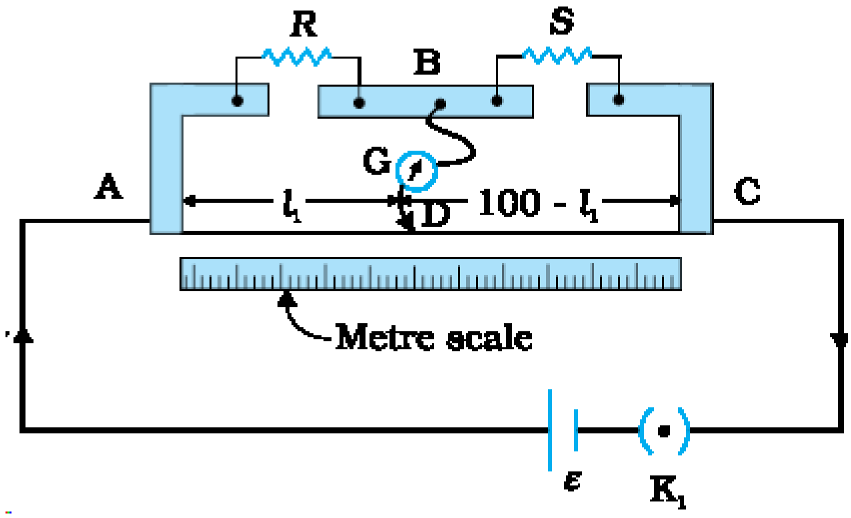
On solving we get,

S=13.5Ω.

From (i) R=$\frac{33.7}{66.3}\square = \frac{33.7}{66.3} \times 13.5$ =6.86Ω

Example

When two known resistances R and S are connected in the left and right gaps of a meter bridge, the balance point is found at a distance $\square_1$ from the zero end of the meter bridge wire. An unknown resistance X is now connected in parallel to the resistance S and the balance point is now found at a distance $\square_2$ from the zero end of the meter bridge wire in figure. Obtain a formula for X in terms of $\square_1$, $\square_2$, □.



Solution

When resistance R and S are connected to the left and right gaps of Meter Bridge and bridge is balanced at length l_1 from zero end, then

$$\frac{\square}{\square} = \frac{\square_1}{100 - \square_1}$$

When unknown resistance X is connected in parallel to S, then effective resistance in right gaps is

S'=$\frac{\square\square}{\square+\square}$

Now, balance point is obtained at length l_2 ,

$$\therefore \frac{\square}{\square'} = \frac{\square_2}{100 - \square_2}$$

$$\frac{\square(\square + \square)}{\square\square} = \frac{\square_2}{100 - \square_2}$$

Putting the value of S', we have:

Dividing (iii) by (i), we get

$$\frac{\square + \square}{\square} = \frac{\square_2}{100 - \square_2} \times \frac{100 - \square_1}{\square_1}$$

Or $\frac{\square}{\square} + 1 = \frac{\square_2(100-\square_1)}{\square_1(100-\square_1)}$

$$\frac{\square}{\square} = \frac{100\square_2 - \square_1\square_2 - 100\square_1 + \square_1\square_2}{\square_1(100-\square_2)} = \frac{100(\square_2 - \square_1)}{\square_1(100-\square_2)}$$

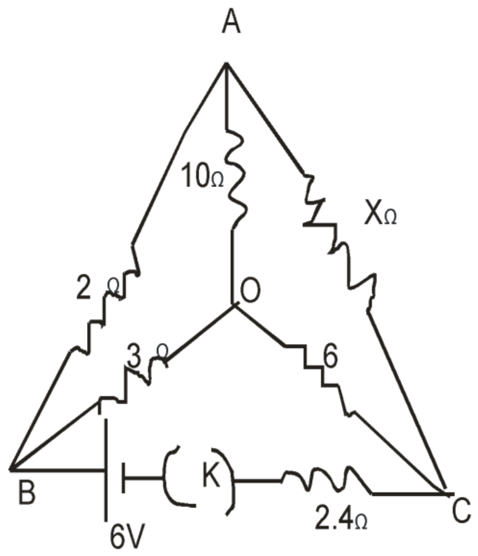
Or

X= $\frac{\square_1(100-\square_2)}{100(\square_2-\square_1)}\square$

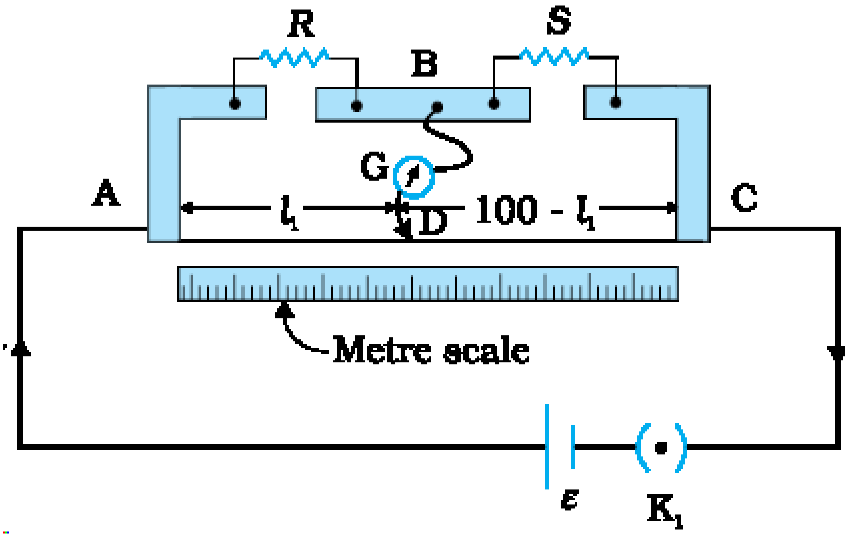
Questions for Practice

- When can you say the Wheatstone bridge is most sensitive?
- How is the balance point affected if positions of galvanometer and battery are interchanged in a meter bridge?
- Why do we prefer to have a balance point in the middle of meter bridge wire?
- Can meter bridge be used for finding the resistance of:
    - Moderate values
    - High values
    - Low values?

    Explain
- Why is the Meter Bridge also called a slide wire bridge?
- When is Wheatstone bridge most sensitive? And why?
- When is a Wheatstone bridge set to be balanced?
- At what position of the jockey on slide Wire Bridge, the results are most accurate?
- Name a practical form of Wheatstone bridge.
- Kirchhoff's first rule obeys the law of conservation of charge. Explain.
- Is it necessary to keep the length of the slide bridge wire 1m? Explain.
- Why should the area of cross-section of the meter bridge wire be uniform? Explain.
- Give the principle of Wheatstone bridge. How do you use it to measure the unknown resistance? Explain with necessary theory
- State and prove principle of Wheatstone bridge Discuss the determination of unknown resistance with its help.
- Find the value of unknown resistance x, in the following circuit, if no current flows through the section AO. Also calculate the current drawn by the circuit from the battery of emf 6V and negligible internal resistance.



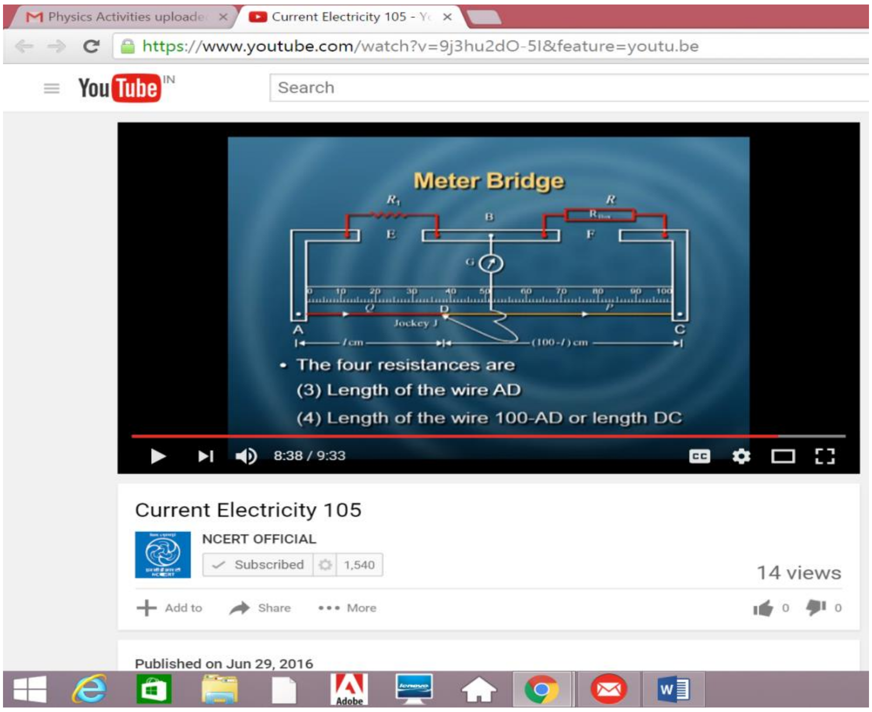
- In a meter bridge, the null point is found at a distance of 40cm from A. If a resistance of 12 Ω is connected in parallel with S, the null point occurs at 50 cm from A. Determine the values of R and S.



Summary

- Kirchhoff's laws are the tools to analyze an electrical circuit. They enable us to find potential drop or current in a branch/device.
- Wheatstone bridge as an assembly of four resistances .

- If a battery is connected across A and C .No current will flow between B and D if $\frac{\square}{\square} = \frac{\square}{\square}$ or if a galvanometer is connected across it would show null or zero deflection.Above condition of balance can be used to determine the resistance, if we know the other three resistances.Bridge is an arrangement in which four electrical devices are connected in the form of a rhombus.Wheatstone bridge is an electrical device used for finding values of unknown resistance.Wheatstone bridge is said to be balanced when the galvanometer shows no deflection, under balanced condition, products of opposite resistances are equal.
- Meter bridge is a modified Wheatstone bridge
- Meter Bridge is designed using a meter long resistance wire mounted on a wooden board. Suitable terminals are placed, to connect one known resistance and an unknown resistance , a battery, a galvanometer and a jockey.Very small, very large resistances cannot be found using meter bridges.Balance point is most affected by interchanging positions of battery and galvanometer.It is desirable to get a balance point in the middle.
- **Formula for the value of unknown resistance**



$$\square_2 = \frac{\square_1(100 - \square\square)}{\square\square}$$

$$\square = \square\frac{100 - \square_1}{\square_1}$$

S is the unknown resistance

R is the known resistance

L_1 is the balance length (AD)

- Methods to solve problems based on Wheatstone bridge and Meter Bridge